NONPLANAR WING CONCEPTS FOR INCREASED AIRCRAFT EFFICIENCY

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1. Summary

Nonplanar wings offer the possibility of reduced drag compared with planar wings of the same span and lift. However, aircraft are not compared on the basis of drag with fixed span and lift, making the integration and assessment of nonplanar wing concepts complex. This paper deals with some of these issues. A brief review of several concepts from winglets to ring wings is followed by a more detailed look at recent ideas and their application to future transport aircraft. Results suggest that potential efficiency gains may be significant, although non-aerodynamic and off-design characteristics are critical in determining the utility of these concepts for transport aircraft.

2. Introduction -- Why Nonplanar Wings?

The vortex drag of commercial aircraft accounts for a large fraction of airplane cruise drag (typically about 40%) and therefore concepts that result in reduction of vortex drag may have a significant effect on fuel consumption, the hundreds of millions of dollars spent annually by airlines on fuel, and its effect on the environment. Vortex drag is even more significant at low speeds where vortex drag typically accounts for 80%-90% of the aircraft's climb drag at critical take-off conditions. Although one might argue that take-off constitutes a very small portion of the flight, its influence on the overall aircraft design is profound. Since conditions associated with engine-out climb shortly after take-off are often critical constraints in the aircraft design, changes in aircraft performance at these conditions influence the overall design and so have an indirect, but powerful, effect on the aircraft cruise performance. While a 1% reduction in drag due to lift might improve the cruise lift-to-drag ratio by 0.4% with a similar effect on range, the improved low speed climb performance may make it possible to achieve acceptable take-off and climb with almost 1% greater take-off weight, leading to an increase in range several times that associated with the simple cruise L/D improvement. As a result, drag due to lift has a much greater significance to aircraft performance than might be inferred simply from the aircraft cruise aerodynamics. Furthermore, even for aircraft that are not constrained by a required climb gradient, lower drag at high lift conditions leads to reduced noise.

Of course, induced drag may be easily reduced by increasing the span of a planar wing. A 10% increase in wing span leads to a 17% reduction in vortex drag at fixed speed and lift. A primary reason that wing spans are not increased to reduce drag is that the higher structural weight and cost make such efforts counterproductive. Nonplanar wing concepts must therefore be assessed similarly, taking into account more than just the potential improvements in cruise aerodynamics.

In fact, some unconventional nonplanar aircraft concepts are promising more because of their structural characteristics than their aerodynamic features. Some designs exploit the nonplanar geometry to improve effective structural depth of the wing system and can achieve drag reduction indirectly by using the improved structural efficiency to accommodate larger spans without higher structural weight. Other design concepts utilize the differences between nonplanar and planar wing load variation with lift coefficient to reduce structural loads at critical conditions and again save in weight or add span at fixed weight.

Finally some nonplanar concepts are motivated by other considerations including the possibility of improved high lift performance or desirable stability and control characteristics.

In this paper we will consider several of these concepts, describe their potential advantages and difficulties, and explore some of the basic ideas that motivate many nonplanar wing designs.

3. What is possible? Assessing the Potential

Before describing specific concepts, it is interesting to evaluate the magnitude of the potential gains associated with nonplanar wings. Do such concepts as offer the potential for 1-2% drag reductions or much larger savings? We start with the simplest analysis possible – considering first just the vortex drag of the wing using the linear aerodynamics and optimal loading ideas described in section 5 and applying this to some classical nonplanar wing concepts.

The minimum vortex drag for systems with the same geometric span and carrying to same total lift is shown in figure 1 for biplanes, boxplanes, a ring-wing, and winglets with varying ratios of height to span. These results were computed using an optimizing vortex lattice code, but agree with classical solutions from Prandtl, von Karman and Burgers, Cone, and Jones. Among the well-known results we note that a ring wing has half of the vortex drag of a monoplane of the same span and lift. A biplane achieves this same drag savings in the limit of very large vertical gap and the boxplane achieves the lowest drag for a given span and height, although winglets are quite similar.

Considerable savings in induced drag are possible for a fixed span if large vertical extents are permitted, with more than 30% drag reductions possible with a height-to-span ratio of 0.2.

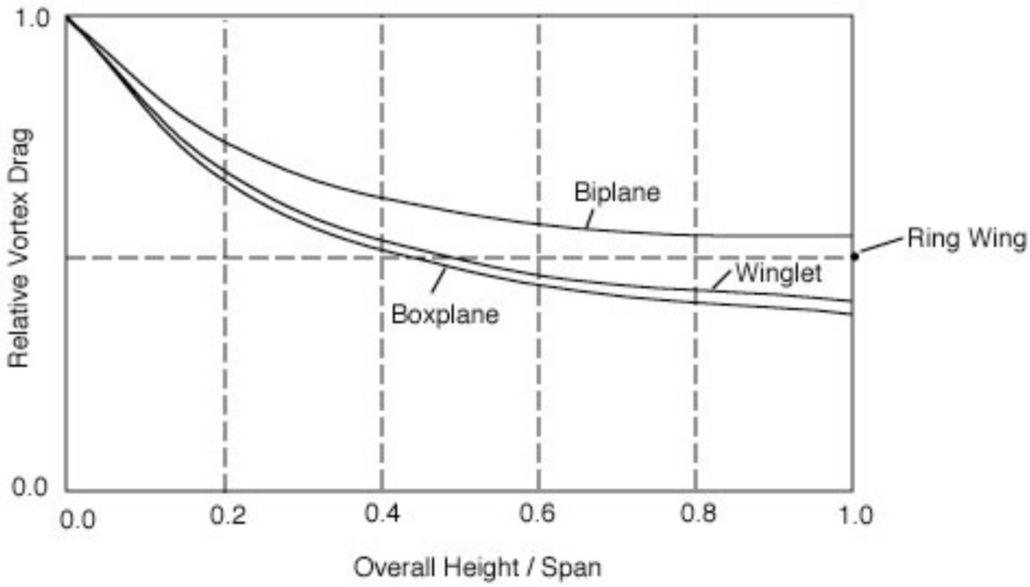


Figure 1. Induced drag variation with allowable height for nonplanar systems

Figure 2 illustrates the span efficiency (induced drag of planar wing / induced drag of the nonplanar system of the same span and lift) for several nonplanar geometries. Each of the geometries is permitted a vertical extent of 20% of the wing span. Each design has the same projected span and total lift. The results were generated by specifying the geometry of the trailing vortex wake and solving for the circulation distribution with minimum drag. So, each of the designs is assumed to be optimally twisted. This was done by discretizing the vortex wake and solving a linear system of equations for minimum drag with a constraint on overall lift as described in section 5. Similar results for a variety of shapes have been described by Cone, Munk, Letcher, Jones, and others.

The results illustrate the variability in span efficiency among these designs. Note that the vertical extent of the system near the tips is the critical parameter and that although the boxplane represents the absolute minimum solution, many other concepts provide very similar drag reductions and show that spanwise camber is most effective near the tip (Lowson 1990).

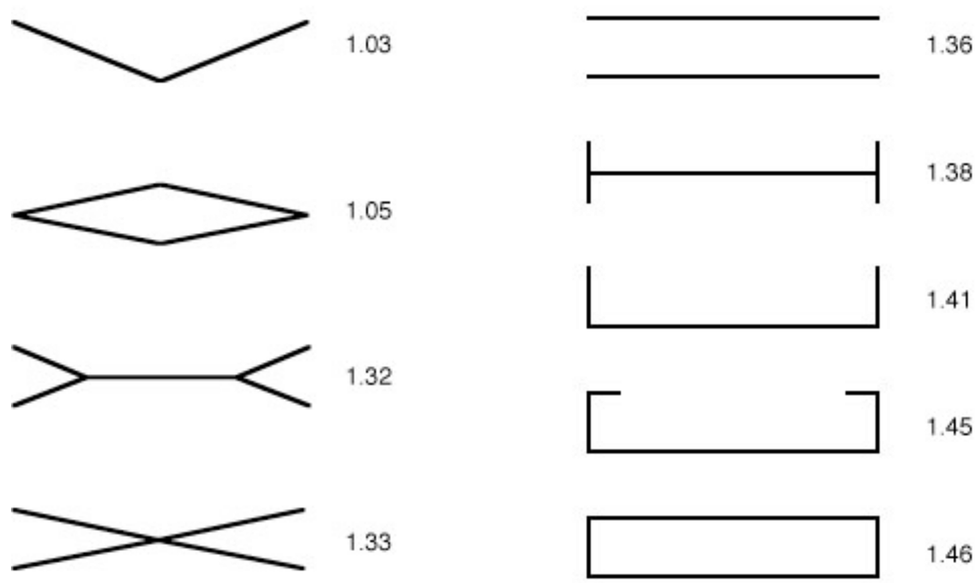


Figure 2. Span efficiency for various optimally loaded nonplanar systems (h/b = 0.2)

Such designs may be of interest because of their potential for lower vortex drag at a fixed span, a key constraint for many aircraft, including very large commercial transport concepts. However, several non-aerodynamic features are of interest as well including effects on stability and control, characteristics of wake vortices, and structural implications of the nonplanar design. These are discussed further in section 4, but even a quick look at some of the fundamental structural implications provides a different perspective on these concepts.

Adding vertical surfaces such as winglets adds wetted area and weight due to higher bending moments. The weight of a cantilevered biplane is increased since for a fixed total area, the chords (and dimensional thickness) of each wing are halved. Figure 3 (from Jones) shows that with fixed integrated bending moment (a rough indicator of wing weight) winglets produce about as much drag savings as planar tip extensions. The maximum induced drag reduction of about 11% compared with an elliptically loaded planar wing is achieved with a 15%-20% span extension or with no span extension and winglets that are 20% of the semi-span in height.

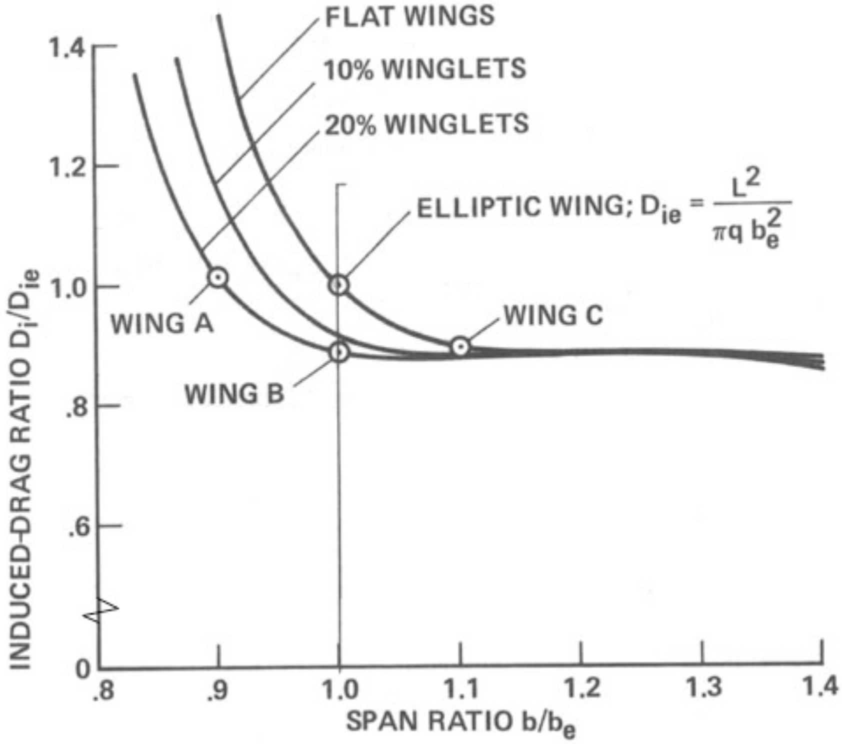


Figure 3. Variation in induced drag for wings and wings with winglets having fixed integrated bending moment.

Of course, increasing span at fixed area, reduces the chord and the structural box height when the airfoil t/c is constrained. This effect is not captured when wings are compared on the basis of root bending moment or integrated bending moment. Thus, when a more realistic weight estimation method is used (see Kroo, 1984 for a first step in this direction), the results are less dramatic. Figure 4 shows the variation in planar wing drag with span at fixed structural weight, indicating a smaller, 4% reduction in vortex drag with nonelliptic loading and increased span. For some applications, the simpler and more encouraging results from figure 1 and 2 are relevant since the aircraft must operate with a span constraint.

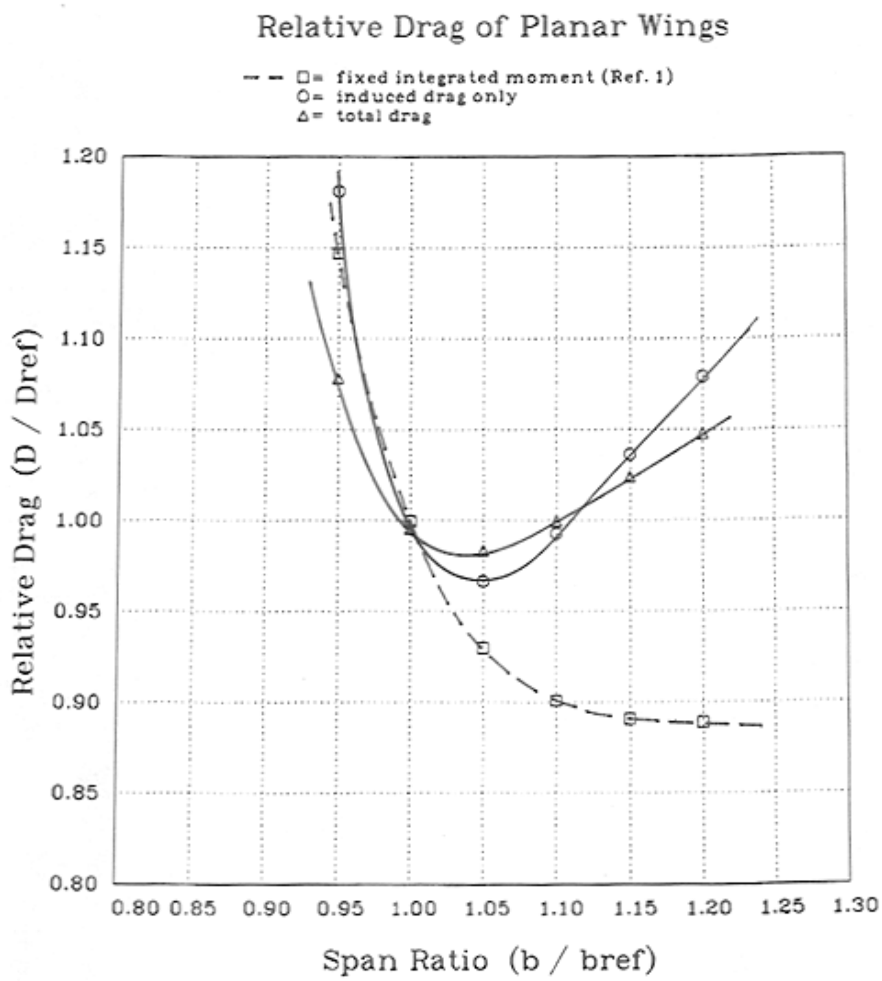


Figure 4. Variation in drag with span with fixed 'structural weight'.

4. Some nonplanar wing concepts.

Nonplanar wings concepts encompass a variety of configurations including biplanes, box-planes, ring-wings, joined wings, and wings with winglets. Apart from configuration differences related to stability and trim, variations in nonplanar geometry represent one of the few major differences in aircraft conceptual design. Nonplanar wing concepts may be divided into a few categories based on their primary geometric or aerodynamic characteristics. Here they are divided into four groups: multiple-wing designs, closed lifting systems, tip devices, and nonlinear aerodynamic concepts.

4.1 Multiple-wing designs

A simple way of creating a nonplanar configuration is to use multiple wings that may not be coplanar. These systems include biplanes and multiplanes and nonplanar formations, from wings in ground effect to swarms.

The multiplane concept was taken to extremes by Phillips in 1904. The aircraft shown below with 20 wings would have had a high span efficiency (though no more than a boxplane of the same span and height), but the very low Reynolds number of each wing would lead to poor performance.

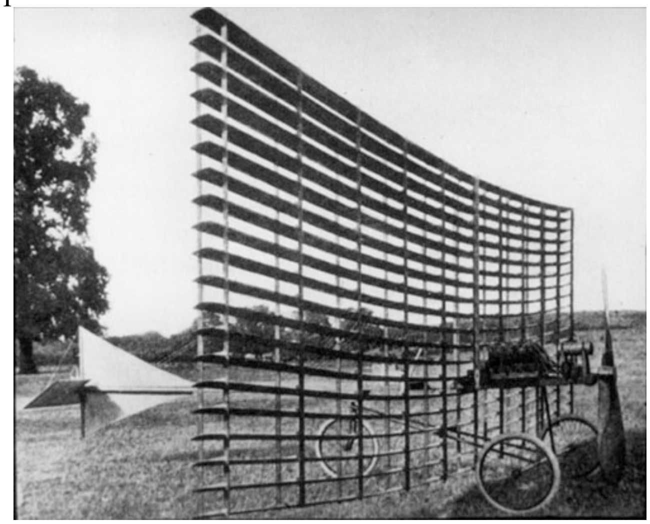


Figure 5. Phillips' Multiplane.

Multiplanes also include biplanes such as the Wright 1902 glider shown below. Although the Wright brothers exploited the structural advantages of biplanes, rather than the lower vortex drag for fixed span and lift, their motivation was partly aerodynamic. Based on their own tests and those of Otto Lilienthal, it was apparent that at very low Reynolds numbers (typical of test conditions used by these pioneers) highly cambered, thin sections performed much better than thicker sections, making the cable-braced Lilienthal designs or the Wright biplane concepts especially attractive. Because of the low flight speeds required for Lilienthal's take-offs and landings and for the power plants available to the Wrights, the designs needed to be light and incorporate large wing areas. This requirement was satisfied well with the biplane configuration.

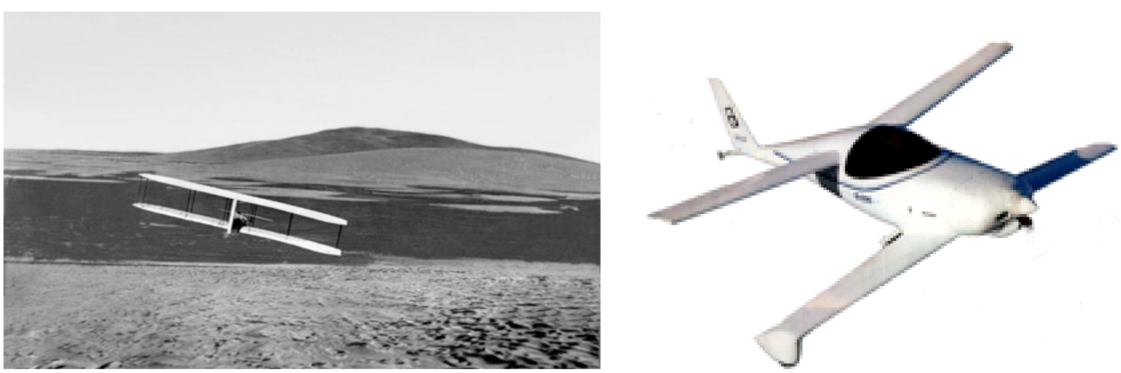


Figure 6. Biplane concepts: Wright 1902 glider and more recent Rutan Quickie.

The struts and cables of early biplane designs also led to large parasite drag, so the effects of improved span efficiency were not obvious. Several modern proposals for cantilevered or semi-cantilevered biplanes have emphasized the lower vortex drag of such configurations at the expense of structural efficiency, Reynolds number, and fuel volume.

The induced drag of a multiplane is lower than that of a monoplane of equal span and total lift because the nonplanar system can influence a larger mass of air, imparting to this air mass a lower average velocity change, and therefore less energy and drag. For a biplane, if the two wings are separated vertically by a very large distance, each wing carries half of the total lift, so the induced drag of each wing is 1/4 that of the single wing. The inviscid drag of the system is then half that of the monoplane.

In addition to the well-known advantages in vortex drag, the favorable interference between two wings of a closely-coupled biplane can be used to improve the section performance. The lower-than-freestream velocity at the trailing edge of the forward wing and the new boundary layer on the downstream wing can be exploited and some of the difficulties with lower Reynolds numbers for the biplane as compared with a monoplane can be alleviated if not turned to advantage. Gains in C_{Lmax}, width of laminar drag bucket, and drag divergence Mach number at fixed t/c are possible with good multiple element section design. As an example, a single fully-laminar section (100% laminar flow on upper and lower surfaces) can support a C_L of about 0.4. A 2-element wing can be designed with an overall C_L of about 0.75. This may help to explain the preference for biplanes in the low Reynolds number world of insects.

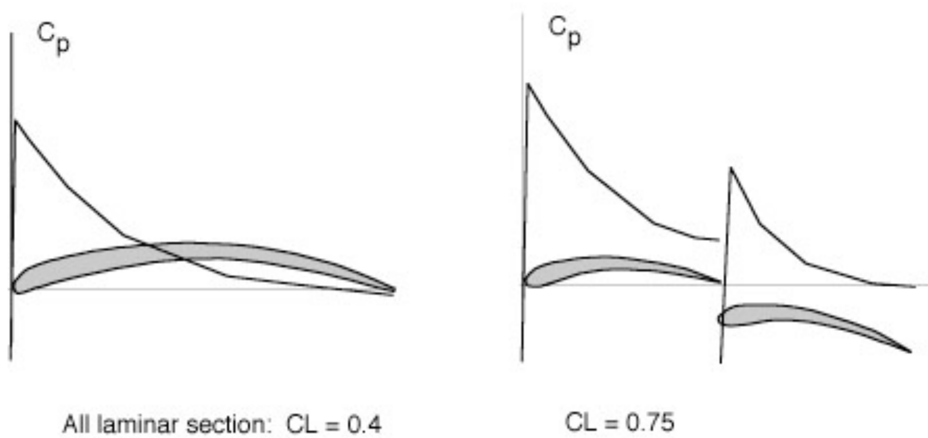


Figure 7. Close coupled nonplanar wings can improve C_{Lmax} by interfering in a manner similar to slotted flaps. This is especially useful at very low Reynolds numbers.

Biplanes represented an early example of nonplanar wings and the potential for induced drag reduction was not lost on early aerodynamicists. By assuming that each wing was elliptically loaded, Prandtl derived a simple expression that was used to determine the induced drag of biplanes with arbitrary vertical gap and span ratio. Prandtl's biplane equation is written:

$$D_i = L_1^2 / q \pi b_1^2 + 2 L_1 L_2 \sigma / q \pi b_1 b_2 + L_2^2 / q \pi b_2^2$$

The interference factor, σ, was computed by integrating the downwash associated with the larger wing over the span of the smaller wing's wake in the Trefftz plane and so was a function only of the span ratio and the vertical gap between the wings. If the two wings had the same span and carried the same lift and were separated by a very large distance, the induced drag of the system approached 50% of the drag of a monoplane with the same total lift and span. As the wings were brought closer together, they would behave as a single wing with an overall span efficiency of 1.0. Interestingly with zero vertical gap or with infinite vertical gap, the optimal loading on each wing of a biplane with $b_1=b_2$ is elliptical. In general we require equal and constant downwash on each wing of the arrangement and so the optimal loading for non-zero, but finite, gaps would not be elliptical. This was of little significance for most biplanes, and tables of σ were used by aircraft designers to estimate the biplane induced drag. The saving could be substantial as suggested by figure 2, although the high parasite drag of struts and cable bracing dominated the drag picture. Now that cantilever structures can be built efficiently, one might ask if a modern, clean biplane arrangement might provide some drag reduction opportunities. This was suggested by Lange (1974) and designs such as Rutan's Quickie (Downie 1984) have achieved some success. However, practical considerations such as fuel volume, structural weight, and lower Reynolds number usually overwhelm potential vortex drag advantages.

A more common example of a modern biplane is the conventional wing plus horizontal tail geometry. Such configurations can be analyzed with Prandtl's biplane equation and this has become a popular approach to the estimation of inviscid trim drag for wing-tail combinations. In the case of a coplanar wing and tail, the vertical gap is 0 and the σ in Prandtl's equation approaches $\sigma = b_2/b_1$, where b_1 is the span of the larger surface. This approach was initially applied to canard configurations as well, often showing large induced drag penalties since stability and trim considerations produced more than optimal lift on the smaller span forward surface. Researchers were surprised to find that in experimental tests canard designs performed much better than predicted (Butler 1982). This was due to the fact that the actual load distribution on the wing of a canard configuration was not elliptical, but rather closer to the optimal load distribution in this case. Modifications to the biplane equation, based on optimal, rather than elliptical loads provided much better comparison with experiments and yielded the correct result that for two coplanar surfaces, the minimum total induced drag depends only on the maximum span and total lift. That the inviscid trim drag for conventional or canard designs could approach zero in theory by the correct choice of wing lift distribution was obvious in view of Munk's stagger theorem. It was also noted that even with elliptic loading, the same cancellation of trim drag could be achieved with a three-surface configuration – a downloaded aft-tail cancelled the wake of the lifting canard. This result led to some configuration studies, but the result in each of these cases was not very robust – small vertical gaps and the initial wake roll-up prevented complete cancellation of wakes and drag rose very quickly with gap in real situations. Moreover, in the canard case, the optimal wing load distribution produced larger root bending moments than the elliptical distribution and when structural weight was included in the optimization, aft tail designs showed advantages for stable 2-surface combinations in terms of total drag (McGeer 1983).

Another biplane-related concept involves the exploitation of ground effect, where the second wing of the biplane is a virtual image of the main wing. Again the Prandtl biplane equation or a variant can be used to evaluate the total induced drag with ½ of the total associated with the actual wing. For most aircraft the importance of ground effect is not great over much of the flight, but aircraft designed to exploit the effect are intriguing. Large transport aircraft utilizing ground effects have been studied by Lockheed Georgia (Lange 1980) and applications to transpacific cargo aircraft have been studied recently. Interesting work on such wing-in-ground effect aircraft in Russia is now becoming better known through technical publications (cf. Besyadovskiy 1993) and conferences devoted to the concept. The idea is exploited by birds including pelicans which achieve large drag reductions by flying at very small distances above the water surface (Hainsworth 1988).

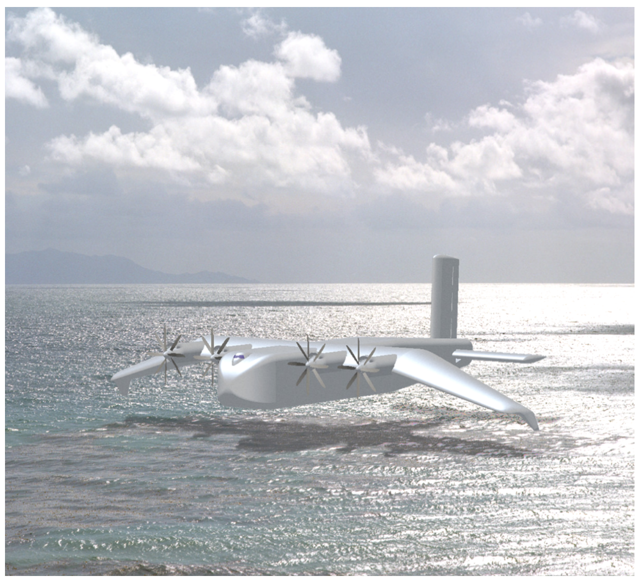


Figure 8. Boeing Pelican conceptual design study for a ground-effect cargo aircraft.

Lastly, multiple, but nonparallel, lifting surfaces may provide some induced drag reduction and favorable interference. Lifting struts have been studied as part of a nonplanar lifting system and provide small potential induced drag benefits, although probably insignificant compared with the drag reduction associated with increased span. Favorable interference between horizontal and vertical tail components are useful mostly to increase lift curve slope, although engine-out vertical tail induced drag can be important and strongly affected by horizontal tail location (Katzoff 1940). Even small devices such as flap hinge covers have been observed to produce a favorable and measurable change in aircraft drag through their interaction with the wing vortex wake. (Page 2000).

4.2 Winglets and Wing Tip Devices

Because of the concentration of vorticity near the wing tip, devices to redistribute and interact with the vorticity in this region have been studied since the introduction of finite wing theory. Although low aspect ratio end plates were originally thought to retard the formation of tip vortices, the operation of such devices is now more commonly understood through the interaction with wake vorticity. The drag reduction is directly related to the shape and extent of the vortex wake. Small tip devices are not able to eliminate or diffuse the wing vortex wake, and, just as for span extensions, the reduction in induced drag is strongly related to the additional bending moments added to the wing.

The results from figure 1 show that a vertical surface located at the wing tip is worth approximately 45% of its height as additional span, if optimally loaded. To produce a large change in the vortex drag without a large increase in wetted area, low aspect ratio endplates were replaced with higher aspect ratio surfaces, termed winglets by Richard Whitcomb, who provided some of the early experimental data and practical design guidelines for such devices (Whitcomb 1976). When the geometric span of the wing is constrained, well-designed winglets do provide significant reductions in airplane drag and have now been incorporated on aircraft ranging from sailplanes to business jets and large commercial transports. The justification for winglets as opposed to span extensions for aircraft that are not explicitly span-limited is less clear. Studies at NASA Langley that compared these two concepts with constrained root bending moment concluded that winglets were to be preferred over span extensions. (Heyson 1977). Studies with constraints on integrated bending moment suggested that the two approaches were almost identical in these respects. (Figure 3, Jones 1980b). A somewhat better weight model (which includes the effects of changes in wing chord on structural efficiency) leads to very similar conclusions as shown in figure 9. The conclusion is that the complexity of the structural model and constraints limits the general applicability of any such conclusions. The evaluation of optimal winglet height and dihedral, depends on the details of the wing structure, whether the wing is gust critical or maneuver critical, whether large regions of the wing are sized based on minimum skin gauge, and whether the design is new or a modification of an existing design. The evaluation of wing tip device advantages must be undertaken for each design and include an array of multidisciplinary considerations. These include the effect on aeroelastic deflections and loads, flutter speed, aircraft trim, stability and control effects (especially lateral characteristics), off-design operation and effects on maximum lift, and finally, marketing considerations.

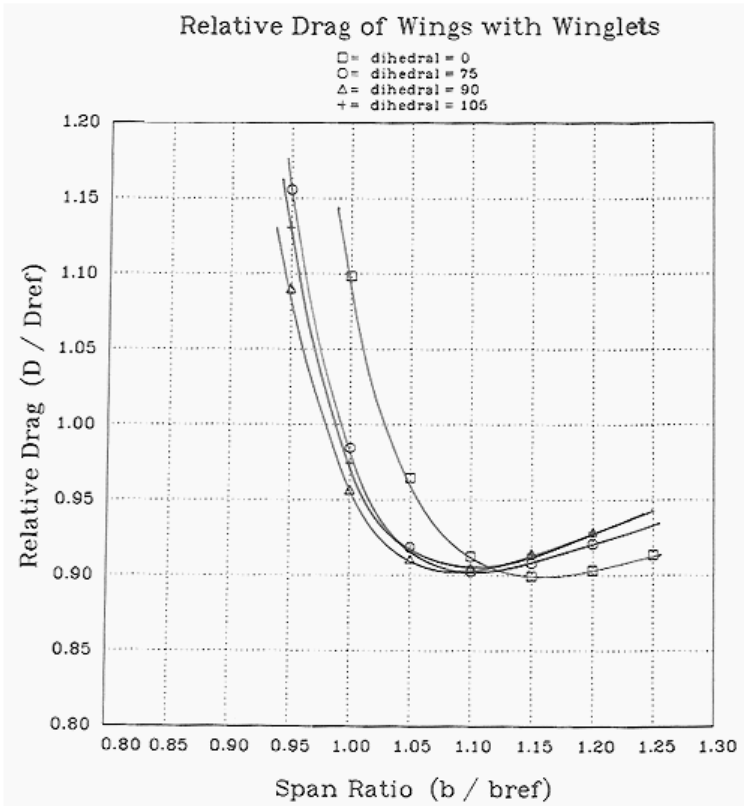


Figure 9. Relative drag of wings with span extensions and winglets of various dihedral angle. Total wetted area is fixed. Taper ratio 0.2, Aspect Ratio 8.

The result of all of these effects have led designers to adopt large winglet surfaces, such as on some canard designs, very small winglets that produce small changes in aircraft properties, but provide a small benefit, or not to use these devices at all. There is no clear answer to the optimal configuration, and even when winglets are adopted, the geometries vary widely (Figure 10). The MD-11 uses a winglet not unlike that described by Whitcomb. The large upper element provides vortex drag reduction without ground clearance problems. Mounted aft on the upper surface, some of the leading edge interaction is reduced – important at high CL conditions, while the small lower winglet can be canted outward, reducing some of the effective dihedral added by the upper element and reducing the torsional inertia which is important in minimizing the impact on flutter speed. The wing tip fences incorporated on the A310 and other Airbus aircraft are very small by comparison, with less impact on the overall aircraft design. These are said to reduce aircraft drag by 1.5% in cruise (Poisson-Quinton 1985).

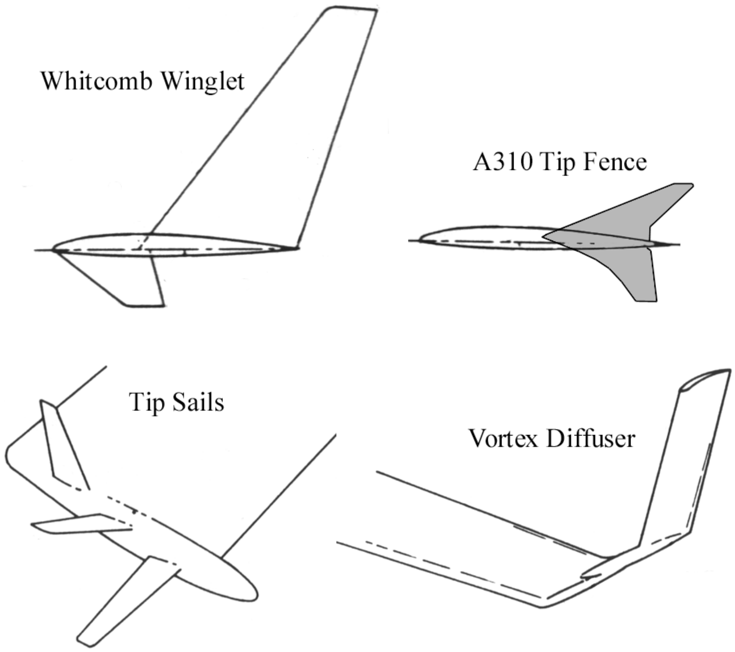


Figure 10. Winglet and wing tip device geometries.

An interesting variant on the winglet concept involves staggering the vertical surface longitudinally. This "vortex diffuser" concept was studied by Lockheed in the 1980's (Hackett 1980) and although it is recognized that the vortex drag reduction is independent of the longitudinal position, some advantages are claimed for this aft positioning of the nonplanar surface. As the vertical surface is moved aft, the effect of the wing on the additional surface increases while the interference on the wing is reduced. This alleviates some of the difficulties with transonic and viscous interactions between winglet and wing, but also points to a limitation with Trefftz plane analysis and the stagger theorem. The minimum induced drag is independent of the longitudinal position of the lifting elements only if the optimal circulation distribution can be achieved. If elements are moved longitudinally, the twist of the surfaces must be changed to maintain the ideal circulation distribution. This may lead to difficulties with off-design operation if the twist is too large, or may in fact be unachievable with any amount of twist.

Wing tip sails (Spillman 1978) are another oft-cited variant of the winglet, consisting of multiple, high aspect ratio lifting elements attached to the wing tip at several dihedral angles. Often analyzed in the near-field, they may also be studied using more conventional far field methods, although, as discussed subsequently, may be more sensitive to wake geometry assumptions. Analyzed using the same Trefftz plane optimization approach as the that used to construct Figure 2, these multiple surfaces appear less effective than a single vertical winglet with the same total span and vertical extent, but may benefit from reduced transonic and viscous interactions at the intersection.

With the myriad of possible geometries for nonplanar wing design one might attempt to find the 'best' shape in a systematic manner. Although the computation of optimal loading is easily accomplished for a given wake geometry using linear theory, the optimization of the wing topology (biplane, spanwise camber, boxplane, winglet, tip sails) is a very difficult problem due to the complexity of the design space and the presence of multiple local minimums. Application of a variable complexity evolutionary algorithm (Gage 1994) produced some interesting results as shown in Figure 11. The system was allowed to build wings of many individual elements with arbitrary dihedral and optimal twist. The figure shows front views of the population of candidate designs as the system evolves along with the best individual from a given generation. The system discovers winglets and then adds a horizontal extension to the winglet, forming a C-like shape. As shown in figure 6, this concept achieves very nearly the maximum induced drag reduction, associated with a boxplane, but eliminates much of the area that would be required to close the box. Further studies showed that the optimal loading on the horizontal extension was downward, reducing root bending moment and providing a positive pitching moment when incorporated on an aft-swept wing. This led to some interesting studies with Boeing on the application of the concept to very large aircraft (McMasters 1998).

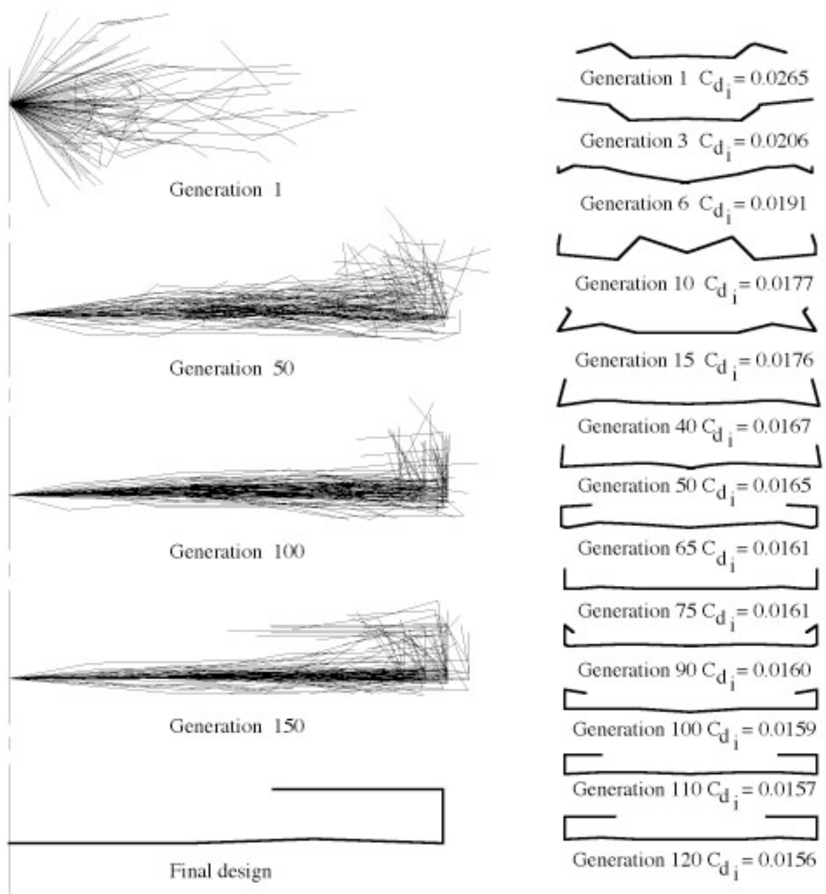


Figure 11. Evolution of C-wing geometry.

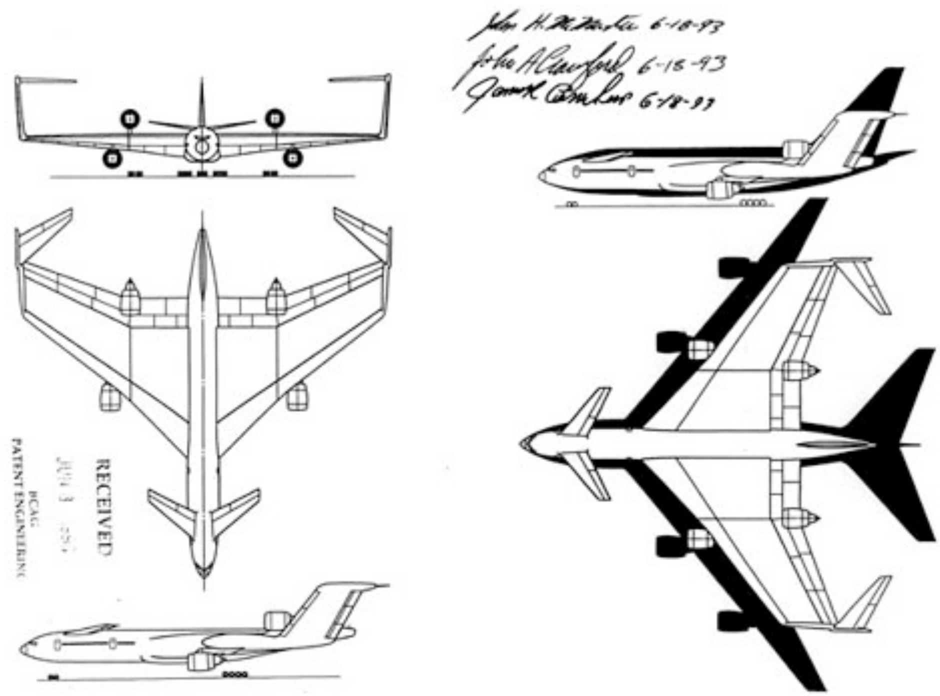


Figure 12. Early conceptual design for a large aircraft with span constraints.

The initial concept arose in response to strict span constraints associated with large aircraft airport compatibility. Incorporating very thick airfoils and accommodating payload partly in the inner wings, the particular design was not pursued much further, but does suggest some interesting variants.

Figure 13 illustrates one of these, in which the added horizontal "winglet-lets" provide efficient nose-up trimming moments and simultaneously increase stability in pitch and yaw.

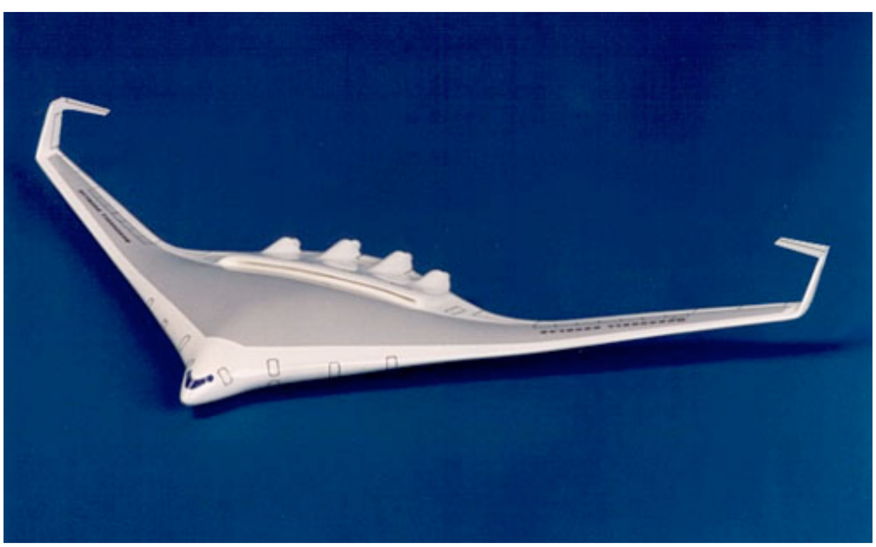


Figure 13. Blended-wing-body concept with C-wing configuration for enhanced stability and control.

The increase in stability provided by the horizontal tip extensions is much larger than might be expected from the planform view since the wing downwash is greatly reduced in this region. This moves the aerodynamic center sufficiently far aft that efficient trim is possible without large sweep and also places the center of additional lift due to flap deflection less far aft relative to the a.c., enabling the use of more effective high lift devices. The reduced sweep also creates greater opportunities for other drag reduction concepts such as laminar flow to be incorporated in this concept.

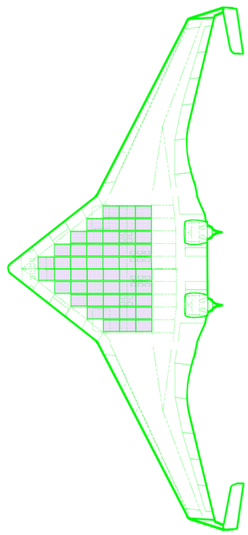
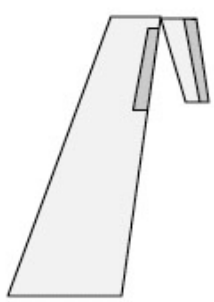


Figure 14. Reduced sweep BWB made possible with C-wing tip geometry offers improved low speed performance and greater potential for laminar flow.

Although the basic C-wing concept was motivated by a span-constrained airplane, the structural characteristics remain important. Any aeroelastician would immediately be struck by the potential flutter penalties associated with the large torsional inertias and coupling. Yet, some possible aeroelastic advantages may be realized with multiple aileron surfaces as shown in figure 15.



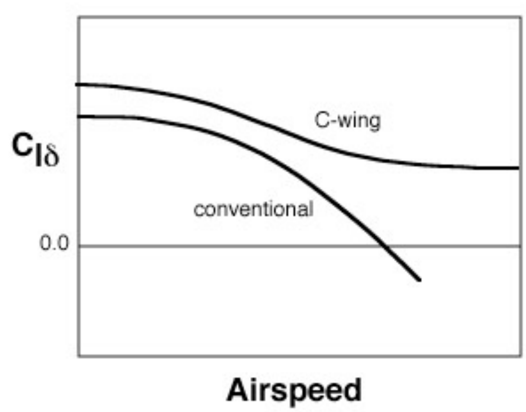


Figure 15. Ailerons on wing and elevons on tip extension could eliminate aileron reversal.

4.3 Closed Systems: Boxplanes, joined wings, lifting struts

One can of course eliminate the wing 'tips' altogether with configurations such as the ring wing (Terry 1964), boxplane (Miranda 1972), joined wing (Wolkovitch 1986), or spiroid tips (Gratzer 1999).

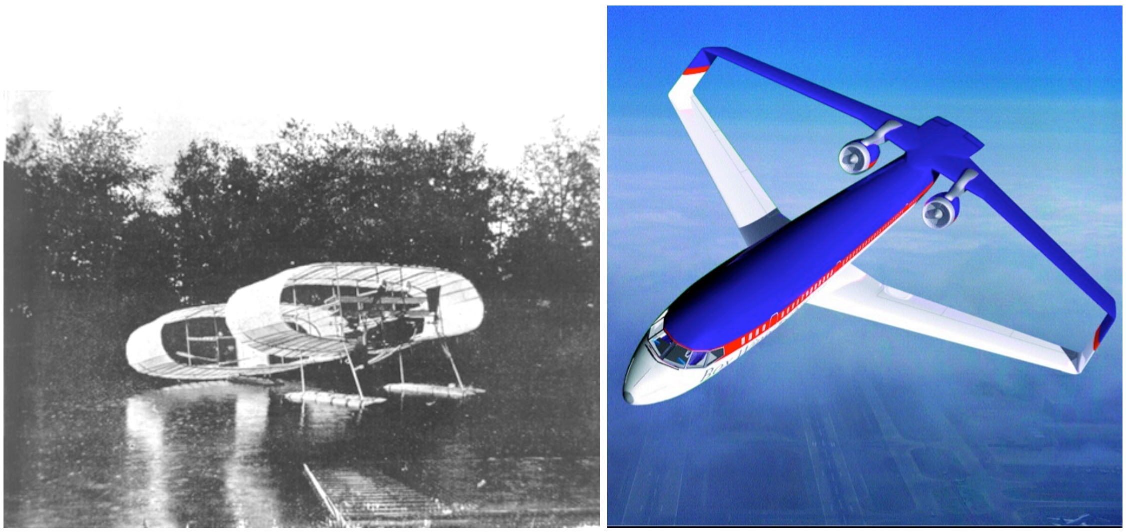


Figure 16. Two concepts for a boxplane design: Bleriot III (left) and Lockheed Boxwing.

While the concept of eliminating the influence of tip vortices with these devices is ill-conceived, such configurations do possess some interesting properties. The boxplane achieves the minimum induced drag for a given lift, span, and vertical extent, and a ring wing or joined wing also can achieve span efficiencies greater than 1 due to their nonplanar geometry, but no particular advantage is seen because these configurations are 'closed'. The one feature that does appear in this case, though, is that the optimal load distribution is not unique. One may superimpose a vortex loop with constant circulation on any of these wing geometries. This changes the local loading, but because the circulation is constant, the wake (and hence the lift and drag) is unchanged. While this does not reduce the vortex drag for a specified lift, it does provide some design flexibility.

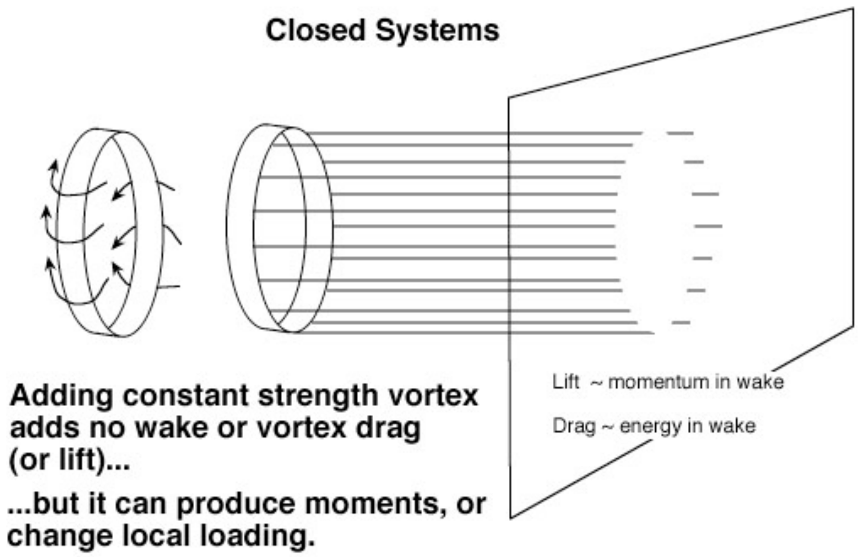


Figure 17. Constant circulation superimposed on a closed lifting system leaves not additional wake, so does not change lift or drag.

The optimal lift distribution of the box plane is generally shown as two horizontal wings that carry the same lift, connected by vertical planes whose circulation goes to zero at their midpoint (von Karman 1935). We can, however, add a fixed circulation to the system so that the lower wing carries the entire lift and the upper wing carries none. The lift and vortex drag are unchanged. This is the reason that the C-wing geometry shown in figure 2 so closely approximates the drag of the boxplane: we simply adjust the constant circulation increment so that the inner part of the upper wing is not needed.

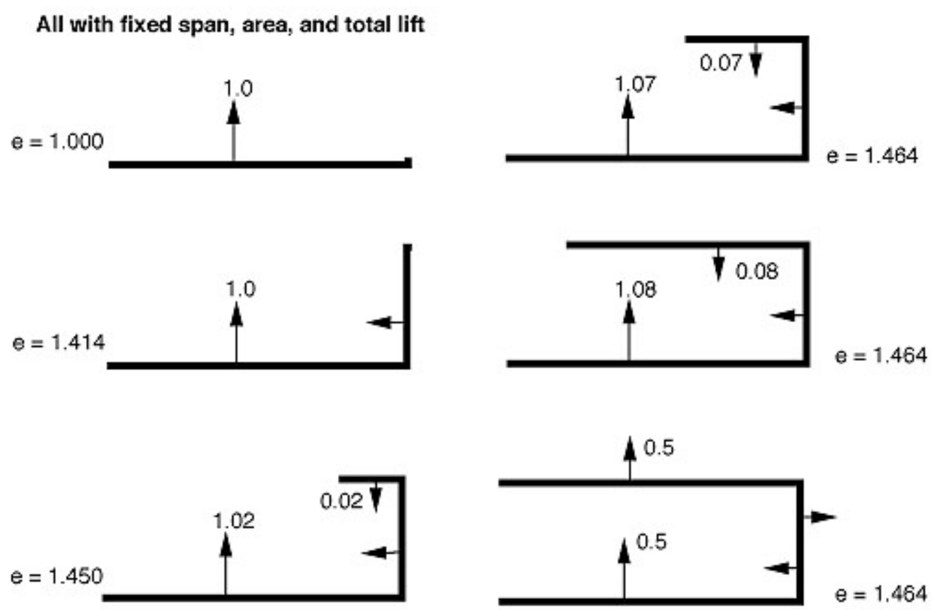


Figure 18. Optimal load distribution on nonplanar wing elements (right side shown).

Figure 19. Joined wing concept gains little induced drag advantage from nonplanar character but can reduce trim drag by exploiting closed lifting system.

When applied to the joined wing configuration, whose upper and lower surfaces are staggered longitudinally, the constant circulation loop can add an arbitrary pitching moment to the design, providing trim over a wide range of lift centroid positions without a vortex drag penalty. When adapted to a winglet or system of wing tip devices -- as on the spiroid tip (Gratzer, 1999) or WingGrid (LaRoche 1998) -- this idea would not seem to provide induced drag advantages at fixed structural weight, but might be used to moderate the local lift coefficients in favorable ways.

Figure 19. Flight test of WingGrid concept.

4.4 Nonlinear aerodynamic considerations

Certain “nonplanar” concepts exploit some more subtle aspects of the aerodynamics to achieve induced drag reduction. Even planar wings can create nonplanar wakes and this can lead to significant changes in vortex drag. A simple high aspect ratio planar wing with a curved trailing edge at angle of attack produces a nonplanar wake as illustrated in figure 20. The effect on induced drag is very small, but the concept can be accentuated.

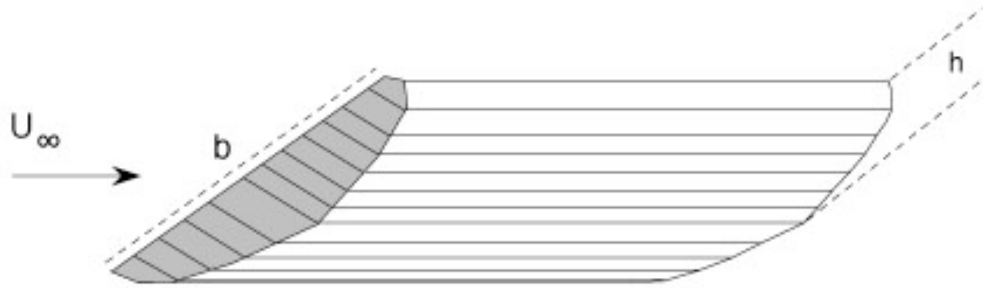


Figure 20. Initial nonplanar wake created by planar wing at angle of attack.

The keel of the recent Swiss entry in the America's Cup yacht race incorporated two wings in tandem (figure 21). The leeway angle of the hull and the motion of the wake of the forward surface leads to the development of an effective vertical gap between the Trefftz plane wakes of the two surfaces and a biplane-like drag reduction. It is noted that these boats were quite fast, but suffered from stability and control problems.

Figure 21. Keel arrangement for recent racing sailboat (Swiss America's Cup entry 1999).

As part of an investigation into these higher order effects on vortex drag, Smith considered an entirely planar wing with multiple elements at the tip (Figure 9, Smith 1996). The idea here was to introduce a wake geometry that looked very much like that associated with tip sails or two winglets with moderate dihedral. The sweep of the wing trailing edges introduced wake dihedral at angle of attack, even when the wake was assumed streamwise. The figure shows the planform shape and the shape of the wake trace when the wing is at 9 degrees incidence and the wake is assumed to be streamwise.

Based on this wake shape, an induced drag savings of about 5% is possible at this condition. Of course the wake does not trail from the wing in the streamwise direction, and careful computation of the rolled-up wake geometry and inviscid drag shows that the effect of wake roll-up is to roughly double the gain expected for the streamwise wake. This 11% increase in span efficiency was significant and the concept was studied in more detail both experimentally and computationally. Figure 9 shows the computed wake geometry and wing paneling used to compute vortex drag with the high-order panel code, A502. The concept was fabricated and tested at NASA's Ames Research Center. Data from the computations, balance, and detailed wake surveys were found to agree well, showing 10%-11% reductions in vortex drag due to these 'higher-order' effects. The results are intriguing, and although the configuration was selected to exaggerate a particular effect rather than to serve as a good airplane wing, its potential application to aircraft is interesting.

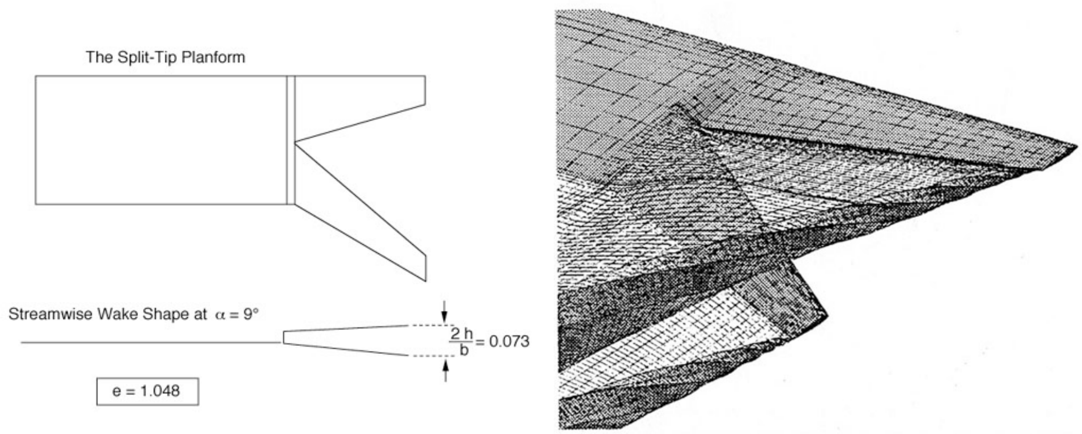


Figure 22. Split tip planform and linear wake projection (left), panel and computed wake geometry (right)

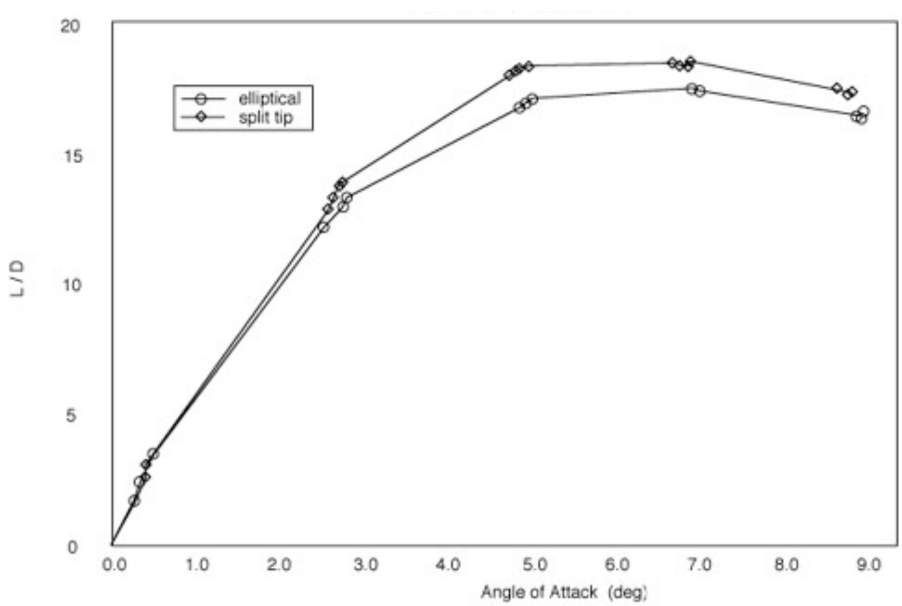


Figure 23. Measured L/D increase due to nonlinear wake effects on split tip.

5. Analysis fundamentals

5.1 Linear analysis and optimal loading

In the case of linear aerodynamic theory, the solutions for minimum vortex drag can be computed in several ways. Often it is convenient to solve directly for the circulation distribution that leads to the minimum drag, subject to bending moment, lift, or trim constraints. When the objective is quadratic and the constraints linear (as they are in several interesting problems), the quadratic programming problem can be solved with a single linear system solution. Alternatively, we may solve for the optimal downwash distribution in the Trefftz plane and then infer the ideal loading based on a Schwartz-Christoffel transformation of the geometry into a simpler system that can then be integrated analytically. The latter approach is that taken by Jones and Lasinski, (1980b) and has certain advantages in understanding the sources of drag reduction. In either case, the basic expression for vortex drag, computed in the far-field is:

$$D_{ind} = \int_{wake} \frac{\rho}{2} \vec{V}_{trefftz} \cdot \hat{n} \gamma(\ell) d\ell$$

where: $V_{Trefftz}$ is the induced velocity in the Trefftz plane. The dot product represents the component of this wake-induced velocity perpendicular to the trace of the wake, here termed normalwash. Since the induced velocity scales linearly with the circulation, the drag may be approximated as a discrete sum as follows:

$$D_{ind} = \frac{\rho}{2} \sum_i \sum_j V_{nij} \gamma_i \gamma_j \triangle \ell_i$$

where Vn is the normalwash. Similarly the lift may be written:

$$L = \sum_i \rho U_\infty \gamma_i \triangle \ell_i \cos \theta_i$$

with θ_i, the dihedral at the i^{th} spanwise station.

The bending-related wing weight may also be related linearly to the circulation at each station (see details in Kroo 84), leading to a quadratic optimization problem with linear constraints. The optimality conditions form a simple linear system whose solution provides the optimal loading and minimum drag. This is the process that leads to the results shown in figures 4 and 10.

A somewhat different approach to the optimal loading problem is based on the method of restricted variations and leads to several insights about nonplanar systems. Munk, Jones, and others used this approach for many induced-drag-related problems. The idea is to consider a small variation in the optimal circulation distribution that satisfies the specified constraints. In the case of a nonplanar wing we consider a small variation $\delta\Gamma_1$ at some location on the wing, and another variation $\delta\Gamma_2$ elsewhere on the wing. To maintain constant lift we require that $\delta\Gamma_1 \cos\theta_1 = -\delta\Gamma_2 \cos\theta_2$. We can write the first-order effect of these perturbations on the system induced drag: $\delta D \sim V_{1n}\, \delta\Gamma_1 + V_{2n}\, \delta\Gamma_2$. In order that the original distribution of circulation represent a minimum drag solution we require: $\delta D = 0$. This means that $V_{1n}\, \delta\Gamma_1 \cos\theta_1 = - V_{2n}\, \delta\Gamma_2 \cos\theta_2$. Adding the constraint on lift leads to $V_{1n} \cos\theta_1 = - V_{2n} \cos\theta_2$. This implies that the downwash in the wake behind an ideally loaded planar wing is constant, as shown by Munk in 1921. With the addition of a root bending moment constraint, we find that the downwash should be distributed linearly over the semispan, while Prandtl's problem with fixed integrated bending moments leads to a parabolic variation of downwash. When the wing is not planar the result implies that the optimal normalwash should vary with the cosine of the dihedral of the wing at that spanwise location. Thus for vertical winglets, the velocity induced by the winglet loading should just cancel the sidewash induced by the wing, implying that the optimally loaded winglet (on an unswept wing at least) should have no net inviscid drag or thrust. One can easily apply this approach to study a wide range of possible nonplanar wing concepts, characterized by the shape of their (assumed streamwise) wake in the Trefftz plane.

5.2 Optimal loading with viscous drag and constrained C_l

It might be noted that these results represent inviscid solutions. Some studies have also included the effect of lift dependent section drag in the optimization (e.g. Rokhsaz 1992, Kroo 1984). While the section polar usually has a small effect on the optimal loading, even constant section drag has quite a different effect for planar and nonplanar wings. If the section profile drag coefficient is considered constant and the section C_l is fixed, the optimal lift distribution for a planar wing remains elliptical and the span grows without bound (or until Reynolds number and structural considerations must be included). For nonplanar wings, however, if the lift and section C_l are fixed, adding nonplanar area adds to the total drag. The optimal lift distribution is then a function of the viscous drag and there is an optimal winglet height, for example. A rather simple modification of the result of the previous section is possible in this case again using the method of restricted variations. With the viscous drag given by: $d_p = q\, C_{dp}\, c$, and the chord set by the specified section C_l: $c = \rho\, U\, \Gamma / q\, C_l$, the condition for minimum drag becomes:

$$\delta D = \rho\, V_{1n}\, \delta\Gamma_1 + \rho\, V_{2n}\, \delta\Gamma_2 + \rho\, U\, (C_{dp}/ C_l)\, \delta\Gamma_1 + \rho\, U\, (C_{dp}/ C_l)\, \delta\Gamma_2 = 0.$$

Substituting the condition for fixed lift, the result is:

$$V_{1n}/U = k \cos\theta_1 - C_{dp}/ C_l$$

Or, the optimal normalwash angle, V_n/U, should be proportional to the cosine of the local dihedral minus an constant offset related to the section drag and design C_l. This tends to unload the winglets and gives a simple way of estimating ideal planform shapes.

5.3 Additional Considerations in Assessing Nonplanar Concepts

Although this discussion has focused on the induced drag of nonplanar lifting systems, some of the most compelling aspects of these systems may be missed entirely by this type of simple analysis.

An initial look at these systems may incorporate simplified structural considerations such as fixed root bending moment or integrated bending moment, or may ignore structures all together. Any results of such analyses may lead to incorrect conclusions about the effectiveness of nonplanar systems. However, even an improved structural model that at least sizes the wing structure may still be insufficient, especially since critical load conditions for nonplanar systems are often quite different from cruise conditions. The critical structural loading may bear little resemblance to the cruise load. For example, the additional loading on winglets (change with angle of attack) does not resemble the additional loading of a planar tip extension. The response to lateral gusts is quite different and may result in winglet loads corresponding to many g's of symmetric acceleration. The downloaded portion of a C-wing reduces root bending moment, but adds to the moment on the outboard wing. Yet as angle of attack is increased, the root bending moment relief is reduced while the outer wing is unloaded.

Perhaps more significant to the structural weight of a nonplanar wing system than the cruise loading or additional load distribution is the opportunity to use the nonplanar geometry constructively as in the case of joined wings or strut-braced wings. The change in structural arrangement so completely changes the trades between span and drag that substantial savings are possible. However, the analysis of such concepts is complex and simple ideas are likely deceptive. The joined wing concept is one such example. Although the potential for structural weight reduction appears significant, in several studies, the additional stiffness required to prevent buckling, the reduction in C_{Lmax} due to reduced tail length, and other complicating factors were enough to compensate for the improved structural arrangement. This conclusion depends on the specific values assumed for required stability margins and field lengths studied for a particular mission, making it dangerous to draw general conclusions about these designs. (See Gallman.)

Similarly, the appeal of nonplanar systems may involve a combination of factors that lead to an improved design – perhaps with less emphasis on vortex drag than on stability and control. Winglets on the SWIFT sailplane (figure 24), for example, not only reduce induced drag on a configuration for which span has a direct effect on usability, but provide directional stability for this tailless design. They also couple with the elevon loading to induce a favorable yawing moment with stick deflection. Finally they move the aerodynamic center aft to permit improved stability with an efficient trimmed load distribution.

Figure 24. The SWIFT sailplane utilizes winglets to reduce induce drag and to augment stability and control characteristics.

6. Conclusions

Although highly nonplanar lifting systems offer the possibility of dramatic reductions in vortex drag for wings with specified lift and span, they are not a panacea. Rather, nonplanar wings likely will provide small improvements in total drag when their effect on wing weight is considered, but may still be worthwhile as increments in performance become progressively more difficult to achieve.

These configurations do provide more substantial performance benefits in some cases. This is especially true when the concept is fully exploited by resizing or even redesigning the aircraft. In addition to reductions in vortex drag, some of configurations mentioned here have desirable effects on structures, stability and control characteristics, vortex wake hazards, and other practical aspects of the design.

Some particularly interesting concepts that warrant further study include the split tip design, which demonstrates that by manipulating the wake shape as well as the wing shape, some of the advantages of nonplanar wings may be enhanced. The C-wing configuration remains an intriguing design concept with many beneficial characteristics when applied to large aircraft or tailless designs. The implications of this approach remain to be more fully explored.

7. Acknowledgments

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